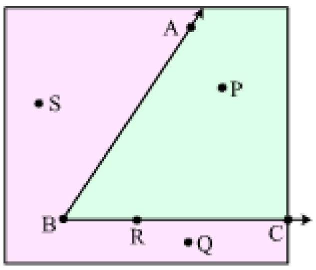
4. Angles and Pairs of Angles

- The given figure represents ∠ABC with some points in its region.



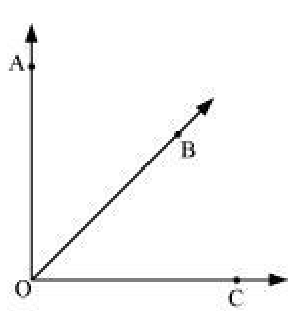
- The region of the angle shaded by green colour lies between the two arms of the angle. This region is called the **interior region of the angle**. Every point in this region is said to lie in the **interior** of the angle. Here, point P is in the interior.

- The region of the angle shaded by pink colour lies outside the two arms of the angle.This region is called the **exterior region of the angle**. Every point in this region is said to lie in the **exterior** of the angle. Here, point Q and S are in the exterior.

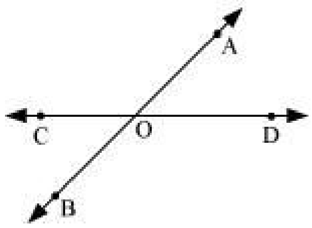
- The boundary of ∠ABC is formed by its arms $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Every point lying on the arms is said to lie on the **boundary of the angle**. Here, points A, B, C and R lie on the boundary of the angle.

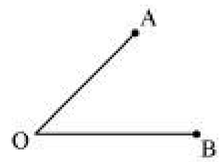
- A pair of angles are called adjacent angles, if:
  - they have a common vertex
  - they have a common arm
  - the non-common arms are on either side of the common arm

    For example, ∠AOB and ∠BOC are adjacent angles as they have a common vertex O, common arm OB, and non-common arms OA and OC lie on either side of OB.

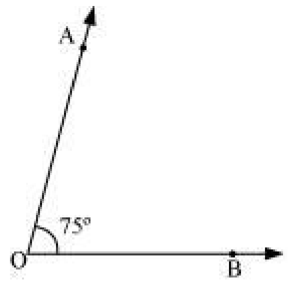


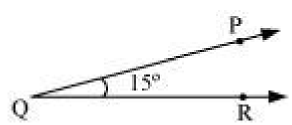
- An angle is made when two lines or line segments meet. For example:





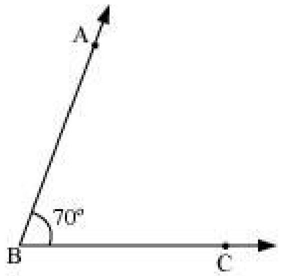
- When the sum of the measures of two angles is 90°, the angles are called **complementary angles**.

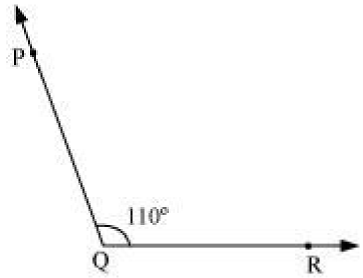




Here, ∠AOB and ∠PQR are complementary as (∠AOB + ∠PQR) = 75° + 15° = 90°.

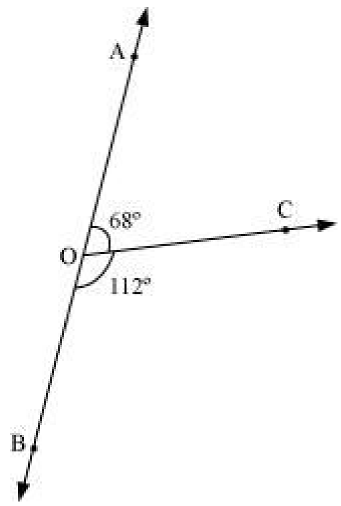
- When the sum of the measures of two angles is 180°, the angles are called **supplementary angles**.





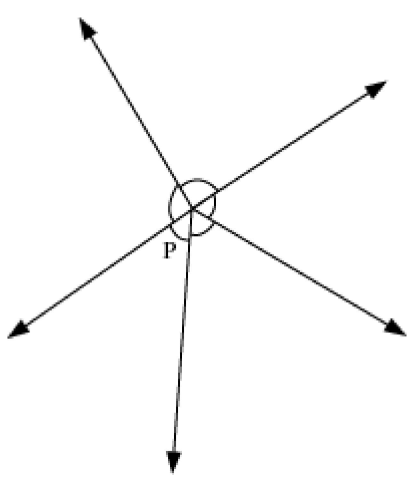
Here, ∠ABC and ∠PQR are supplementary as (∠ABC + ∠PQR) = 110° + 70° = 180°.

- A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays.
- The sum of the measures of the adjacent angles is 180°.



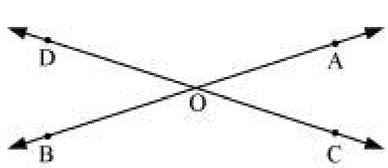
Here, ∠AOC and ∠BOC form a linear pair as ∠AOC + ∠BOC = 180°.

- The sum of angles around a point is equal to 360°.



In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

- When two lines intersect, the vertically opposite angles so formed are equal.



Here, ∠AOC = ∠BOD and ∠AOD = ∠BOC.

- The sum of all the interior angles of an n-sided polygon is given by, $(n - 2) \times 180°$.

Example: What is the number of sides of a polygon whose sum of all interior angles is 720°?

Solution: It is known that,

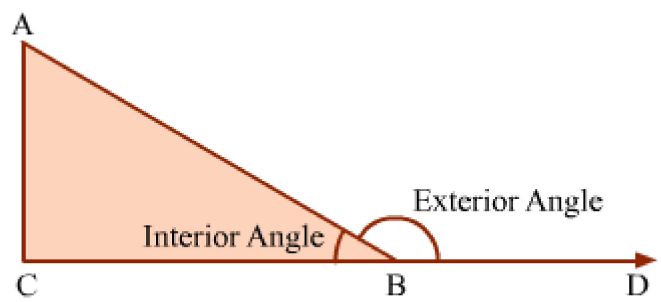
$(n-2)180° = 720°$

$\Rightarrow (n-2) = \frac{720°}{180°} = 4$

$\Rightarrow n = 6$

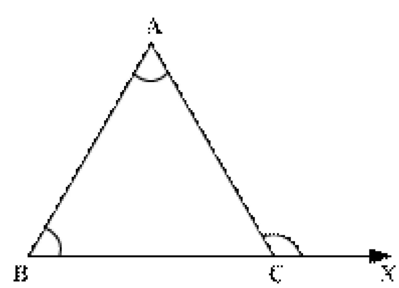
Thus, the required polygon is six-sided.

- The angle formed by a side of a triangle with an extended adjacent side is called an **exterior angle of the triangle**.



It can be seen that in ΔABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, **∠ABD is an exterior angle of ΔABC.**

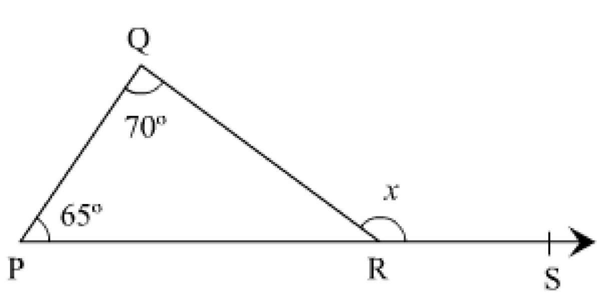
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.



∠ACX = ∠BAC + ∠ABC
This property is known as exterior angle property of a triangle.

Example:

Find the value of x in the following figure.



Solution:

∠QRS is an exterior angle of ΔPQR. It is thus equal to the sum of its interior opposite angles.

$\therefore \angle QRS = \angle QPR + \angle PQR$

$\Rightarrow x = 65° + 70° = 135°$

Thus, the value of x is 135°.

- Two exterior angles can be drawn at each vertex of triangle. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.